

Chapter 4 Lesson 3

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS

Lesson Objectives:

- Multiply and divide mixed numbers
- Translate phrases to expressions with fractions
- Simplify complex fractions
- Simplify expressions written with a fraction bar

Multiply & Divide Mixed Numbers *notes*

Multiply & Divide Mixed Numbers

In the last section we talked about how to multiply and divide fractions. In this section, we will learn how to multiply and divide mixed numbers.

Lucky for us, it's super easy and almost the same as multiplying and dividing regular fractions. The only difference is that first you must convert your mixed numbers to improper fractions.

HOW TO MULTIPLY OR DIVIDE MIXED NUMBERS

Step 1: Convert the mixed numbers to improper fractions.

Step 2: Follow the rules for fraction multiplication or division.

Step 3: Simplify if possible.

Example Problems

$$2\frac{4}{5} \left(1\frac{7}{8}\right)$$

$$3\frac{4}{7} \div 5$$

$$2\frac{1}{2} \div 1\frac{1}{4}$$

Multiply & Divide Mixed Numbers *your turn*

Multiply or divide and write your answer in simplified form.

$$5\frac{4}{5} \left(-2\frac{3}{8}\right)$$

$$-4\frac{4}{5} \left(4\frac{1}{6}\right)$$

$$4\frac{3}{8} \div 6$$

$$2\frac{3}{8} \div 4$$

$$3\frac{2}{3} \div 1\frac{1}{3}$$

$$4\frac{3}{4} \div 1\frac{1}{2}$$

Translate Phrases to Expressions with Fractions *notes*

Translate Phrases to Expressions with Fractions

There are two words that are often used to describe fractions: *ratio* and *quotient*. A quotient is the result of division, and we say "the quotient of a and b is the result you get from dividing a by b, or $\frac{a}{b}$."

Let's practice translating some phrases into algebraic expressions using these terms.

Example Problems

Translate each phrase into an algebraic expression.

The quotient of $4x$ and 8

The quotient of the sum of a and b , and c .

your turn

The quotient of $10s$ and 15

The quotient of $5h$ and 8

The quotient of the difference of x and y , and zw .

The quotient of the difference of a and b , and c .

Simplify Complex Fractions *notes*

Simplify Complex Fractions

We've talked about proper fractions, improper fractions, and mixed numbers. There's one more type of fraction we will talk about: complex fractions!

A complex fraction is a fraction in which the numerator or the denominator contain a fraction.

Here are some examples: $\frac{6}{\frac{7}{3}}$ $\frac{\frac{3}{5}}{\frac{4}{8}}$

They look scary, but they are rather easy to solve! Just remember that the fraction bar means division. That means that $\frac{\frac{3}{5}}{\frac{4}{8}} = \frac{3}{5} \div \frac{4}{8}$, and you already know how to solve those!

Example Problems

Simplify.

$$\frac{\frac{3}{5}}{\frac{4}{8}}$$

$$-\frac{6}{\frac{7}{3}}$$

$$\frac{\frac{x}{z}}{\frac{xy}{4}}$$

$$\frac{2\frac{3}{4}}{\frac{1}{8}}$$

Simplify a Complex Fraction *your turn*

Multiply or divide and write your answer in simplified form.

$$\frac{\frac{2}{5}}{\frac{6}{11}}$$

$$\frac{\frac{3}{7}}{\frac{5}{11}}$$

$$-\frac{7}{8} \div \frac{1}{4}$$

$$-\frac{3}{9} \div \frac{3}{10}$$

$$\frac{\frac{a}{10}}{\frac{ab}{5}}$$

$$\frac{\frac{x}{2}}{\frac{xy}{10}}$$

$$\frac{\frac{5}{8}}{\frac{1}{2} \div \frac{1}{3}}$$

$$\frac{\frac{8}{5}}{3 \div \frac{1}{5}}$$

Simplify Expressions with a Fraction Bar *notes*

Where Does the Negative Go?

One thing that often comes up when talking about fractions is the negative. Where does it go? What if there is more than one negative?

Remember that fractions represent division. The fraction $-\frac{6}{7}$ could be the result of dividing $\frac{-6}{7}$ or $\frac{6}{-7}$. When the numerator and denominator have different signs, the quotient (entire fraction) is negative because fractions are division! If the numerator AND the

denominator have the same sign, then the fraction itself is positive because you are essentially dividing a negative by a negative.

Pretty cool, right?

$$\frac{-1}{4} = -\frac{1}{4}$$

$$\frac{1}{-4} = -\frac{1}{4}$$

$$\frac{-1}{-4} = \frac{1}{4}$$

Expressions with a Fraction Bar

When simplifying expressions with a fraction bar, the fraction bars act as grouping symbols. Imagine that everything in the numerator is in parentheses and everything in the denominator is in parentheses, and simplify the numerator, then simplify the denominator.

$\frac{4 + 8}{5 - 3}$ is the same as $(4 + 8) \div (5 - 3)$.

Let's try some!

Example Problems

Simplify.

$$\frac{4 - 2(3)}{2^2 + 2}$$

$$\frac{(8 - 4)^2}{8^2 - 4^2}$$

Simplify Expressions with a Fraction Bar *your turn*

Simplify.

$$\frac{4 + 6}{11 - 3}$$

$$\frac{6 - 3(5)}{3^2 - 3}$$

$$\frac{(11 - 6)^2}{11^2 - 7^2}$$

$$\frac{(6 - 2)^2}{6^2 - 2^2}$$

EXAMPLE:

$$\frac{[4(-3) + 6(-2)]}{3(2) - 2}$$

$$\frac{-8(2) + 4(-3)}{2(-5) - (-3)}$$

$$\frac{-7(1) - (-9)(-3)}{3(-5) - 2}$$

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS *Practice*

Simplify.

1. $4\frac{5}{8} \cdot \frac{4}{15}$

2. $\frac{15}{22} \cdot 3\frac{4}{5}$

3. $5\frac{1}{3} \div 8$

4. $-12 \div \left(-3\frac{3}{11}\right)$

5. $-9\frac{3}{5} \div 1\frac{2}{5}$

6. $-18\frac{3}{4} \div 3\frac{3}{4}$

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS *Practice*

Translate each phrase into an algebraic expression.

7. The quotient of $6x$ and 11 8. The quotient of r and the sum of b and 9

Simplify.

8. $\frac{\frac{2}{3}}{\frac{4}{9}}$

9. $\frac{\frac{8}{12}}{\frac{-12}{35}}$

10. $\frac{\frac{4}{5}}{\frac{8}{3}}$

11. $\frac{\frac{m}{6}}{\frac{x}{3}}$

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS *Practice*

Simplify.

12. $\frac{4 + 11}{8}$

13. $\frac{22 + 8}{10}$

14. $\frac{15 - (-9)}{18 + 12}$

15. $\frac{6 \cdot 2}{4 \cdot 8}$

16. $\frac{12 \cdot 9 - 3^2}{6 \cdot 9}$

17. $\frac{9(8 - 2) - 3(15 - 7)}{6(4 + 2) - 3(17 - 9)}$

Answer Key

Multiply & Divide Mixed Numbers *notes*

Multiply & Divide Mixed Numbers

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HOW TO MULTIPLY OR DIVIDE MIXED NUMBERS

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Step 2: Follow the rules for fraction multiplication or division.

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Example Problems

$$2\frac{4}{5} (1\frac{7}{8})$$

$$\frac{\cancel{7}^1}{\cancel{5}_1} \cdot \frac{\cancel{15}^3}{\cancel{8}_4}$$

$$\frac{21}{4} = 5\frac{1}{4}$$

$$3\frac{4}{7} \div 5$$

$$\frac{\cancel{25}^5}{\cancel{7}_1} \div 5$$

$$\frac{\cancel{25}^5}{\cancel{7}_1} \cdot \frac{1}{\cancel{5}_1} = 5\frac{5}{7}$$

$$2\frac{1}{2} \div 1\frac{1}{4}$$

$$\frac{\cancel{5}^1}{\cancel{2}_1} \div \frac{\cancel{5}^1}{\cancel{4}_2}$$

$$\frac{\cancel{5}^1}{\cancel{2}_1} \cdot \frac{\cancel{4}^2}{\cancel{5}_1} = 2$$

Multiply & Divide Mixed Numbers *your turn*

Multiply or divide and write your answer in simplified form.

$$5\frac{4}{5} \left(-2\frac{3}{8}\right)$$

$$\frac{29}{5} \cdot -\frac{19}{8} = \left(\frac{551}{40}\right)$$

$$-4\frac{4}{5} \left(4\frac{1}{6}\right)$$

$$-\frac{24}{5} \cdot \frac{25}{6} = -20$$

$$4\frac{3}{8} \div 6$$

$$\frac{27}{8} \cdot \frac{1}{6} = \left(\frac{9}{16}\right)$$

$$2\frac{3}{8} \div 4$$

$$\frac{19}{8} \cdot \frac{1}{4} = \left(\frac{19}{32}\right)$$

$$3\frac{2}{3} \div 1\frac{1}{3}$$

$$\frac{11}{3} \div \frac{4}{3} = \left(\frac{11}{4}\right)$$

$$4\frac{3}{4} \div 1\frac{1}{2}$$

$$\frac{19}{4} \div \frac{3}{2} = \left(\frac{19}{6}\right)$$

Translate Phrases to Expressions with Fractions *notes***Translate Phrases to Expressions with Fractions**

There are two words that are often used to describe fractions: *ratio* and *quotient*. A quotient is the result of division, and we say "the quotient of a and b is the result you get from dividing a by b, or $\frac{a}{b}$."

Let's practice translating some phrases into algebraic expressions using these terms.

Example Problems

Translate each phrase into an algebraic expression.

The quotient of $4x$ and 8

$$\frac{4x}{8} = \frac{x}{2}$$

The quotient of the sum of a , b , and c .

$$\frac{a+b}{c}$$

your turn

The quotient of $10s$ and 15

$$\frac{10s}{15} = \frac{2s}{3}$$

The quotient of $5h$ and 8

$$\frac{5h}{8}$$

The quotient of the difference of x , y , and zw .

$$\frac{x-y}{zw}$$

The quotient of the difference of a , b , and c .

$$\frac{a-b}{c}$$

Simplify Complex Fractions *notes*

Simplify Complex Fractions

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Example Problems

Simplify.

$$\frac{\frac{3}{4}}{\frac{5}{8}} = \frac{3}{4} \div \frac{5}{8}$$

$$\frac{3}{4} \cdot \frac{8}{5} = \frac{24}{20}$$

$$\frac{6}{5}$$

$$\frac{\frac{3}{8}}{\frac{5}{4}}$$

$$\frac{-\frac{6}{7}}{3} = -\frac{6}{7} \div 3$$

$$-\frac{6}{7} \cdot \frac{1}{3} = \frac{-2}{7}$$

$$\frac{6}{\frac{7}{3}}$$

$$\frac{\frac{x}{2}}{\frac{xy}{4}} = \frac{x}{2} \div \frac{xy}{4}$$

$$\frac{x}{2} \cdot \frac{4}{xy} = \frac{2}{y}$$

$$\frac{\frac{x}{2}}{\frac{xy}{4}}$$

$$\frac{\frac{11}{4}}{\frac{1}{8}} = \frac{11}{4} \div \frac{1}{8}$$

$$\frac{11}{4} \cdot \frac{8}{1} = 22$$

$$\frac{2\frac{3}{4}}{\frac{1}{8}}$$

Simplify a Complex Fraction *your turn*

Multiply or divide and write your answer in simplified form.

$$\frac{2}{5} \div \frac{2}{5} = \frac{2}{5} \cdot \frac{5}{2} = \frac{10}{10} = 1$$

$$\frac{3}{7} \div \frac{5}{11} = \frac{3}{7} \cdot \frac{11}{5} = \frac{33}{35}$$

$$-\frac{7}{8} \div -4 = -\frac{7}{8} \cdot \frac{1}{-4} = \frac{-7}{-32} = \frac{7}{32}$$

$$-\frac{3}{9} \div \frac{10}{1} = -\frac{3}{9} \cdot \frac{1}{10} = -\frac{3}{90} = -\frac{1}{30}$$

$$\frac{a}{10} \div \frac{a}{5} = \frac{a}{10} \cdot \frac{5}{a} = \frac{5a}{10a} = \frac{1}{2}$$

$$\frac{x}{xy} \div \frac{xy}{10} = \frac{x}{xy} \cdot \frac{10}{xy} = \frac{10x}{x^2y^2} = \frac{10}{xy}$$

$$\frac{12}{3} \div \frac{8}{5} = \frac{12}{3} \cdot \frac{5}{8} = \frac{60}{24} = \frac{5}{2}$$

$$\frac{8}{5} \div \frac{17}{5} = \frac{8}{5} \cdot \frac{5}{17} = \frac{8}{17}$$

Simplify Expressions with a Fraction Bar *notes*

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denominator have the same sign, then the fraction itself is positive because you are essentially dividing a negative by a negative.

Pretty cool, right?

$$\frac{-1}{4} = -\frac{1}{4}$$

$$\frac{1}{-4} = -\frac{1}{4}$$

$$\frac{-1}{-4} = \frac{1}{4}$$

Expressions with a Fraction Bar

When simplifying expressions with a fraction bar, the fraction bars act as grouping symbols. Imagine that everything in the numerator is in parentheses and everything in the denominator is in parentheses, and simplify the numerator, then simplify the denominator.

$\frac{4+8}{5-3}$ is the same as $(4+8) \div (5-3)$.

Let's try some!

Example Problems

Simplify.

$$\frac{4-2(3)}{2^2+2}$$

$$\frac{4-6}{4+2} = \frac{-2}{6} = \left(-\frac{1}{3}\right)$$

$$\frac{(8-4)^2}{8^2-4^2}$$

$$\frac{4^2}{64-16}$$

$$\frac{16}{48} = \left(\frac{1}{3}\right)$$

Simplify Expressions with a Fraction Bar *your turn*

Simplify.

$$\frac{4 + 6}{11 - 3}$$

$$\frac{10}{8} = \left(\frac{5}{4}\right)$$

$$\frac{6 - 3(5)}{3^2 - 3}$$

$$\frac{6 - 15}{9 - 3} = \frac{-9}{6} = \left(\frac{-3}{2}\right)$$

$$\frac{(11 - 6)^2}{11^2 - 7^2} \quad \frac{5^2}{121 - 49}$$

$$\left(\frac{25}{72}\right)$$

$$\frac{(6 - 2)^2}{6^2 - 2^2}$$

$$\frac{4^2}{36 - 4} = \frac{16}{32} = \left(\frac{1}{2}\right)$$

EXAMPLE:

$$\frac{[4(-3) + 6(-2)]}{3(2) - 2} \quad \frac{-12 + (-12)}{6 - 2}$$

$$\frac{-24}{4} = (-6)$$

$$\frac{-8(2) + 4(-3)}{2(-5) - (-3)}$$

$$\frac{-16 - 12}{-10 + 3} = \frac{-28}{-7} = (4)$$

$$\frac{-7(1) - (-9)(-3)}{3(-5) - 2}$$

$$\frac{-7 - 27}{-15 - 2} = \frac{-34}{-17} = (2)$$

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS Practice

Simplify.

1. $4\frac{5}{8} \cdot \frac{4}{15}$

$\frac{37}{8} \cdot \frac{4}{15}$

$\frac{37}{30}$

2. $\frac{15}{22} \cdot 3\frac{4}{5}$

$\frac{15}{22} \cdot \frac{19}{5} = \frac{57}{22}$

3. $5\frac{1}{3} \div 8$

$\frac{16}{3} \cdot \frac{1}{8}$

$\frac{2}{3}$

4. $-12 \div (-3\frac{3}{11})$

$-12 \div \frac{-36}{11}$

$\frac{1}{12} \cdot \frac{11}{36} = \frac{11}{3}$

5. $-9\frac{3}{5} \div 1\frac{2}{5}$

$-\frac{48}{5} \div \frac{7}{5} = -\frac{48}{5} \cdot \frac{5}{7}$

$-\frac{48}{7}$

6. $-18\frac{3}{4} \div 3\frac{3}{4}$

$-\frac{75}{4} \div \frac{15}{4}$

$-\frac{5}{4} \cdot \frac{4}{15}$

-5

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS Practice

Translate each phrase into an algebraic expression.

7. The quotient of $6x$ and 11

$$\frac{6x}{11}$$

8. The quotient of r and the sum of b and 9

$$\frac{r}{b+9}$$

Simplify.

8. $\frac{2}{9} \div \frac{3}{4}$

$\frac{2}{9} \div \frac{3}{4} = \frac{2}{9} \cdot \frac{4}{3} = \frac{8}{27}$

9. $\frac{8}{12} \div \frac{12}{35}$

$\frac{8}{12} \div \frac{12}{35} = \frac{8}{12} \cdot \frac{35}{12} = \frac{280}{144} = \frac{35}{18}$

10. $\frac{5}{8} \div \frac{1}{2}$

$\frac{5}{8} \div \frac{1}{2} = \frac{5}{8} \cdot \frac{2}{1} = \frac{10}{8} = \frac{5}{4}$

11. $\frac{6}{3} \div \frac{m}{2x}$

$\frac{6}{3} \div \frac{m}{2x} = \frac{2}{1} \cdot \frac{2x}{m} = \frac{4x}{m}$

MULTIPLY & DIVIDE MIXED NUMBERS & COMPLEX FRACTIONS Practice

Simplify.

12. $\frac{4 + 11}{8}$

$$\frac{15}{8}$$

13. $\frac{22 + 8}{10}$

$$\frac{30}{10} = 3$$

14. $\frac{15 - (-9)}{18 + 12}$

$$\frac{15 + 9}{30} = \frac{24}{30} \div 6 = \frac{4}{5}$$

15. $\frac{6 \cdot 2}{4 \cdot 8}$

$$\frac{3}{8}$$

16. $\frac{12 \cdot 9 - 3^2}{6 \cdot 9}$

$$\frac{108 - 9}{54} = \frac{99 \div 9}{54 \div 9} = \frac{11}{6}$$

17. $\frac{9(8 - 2) - 3(15 - 7)}{6(4 + 2) - 3(17 - 9)}$

$$\frac{9(6) - 3(8)}{6(6) - 3(8)} = \frac{54 - 24}{36 - 24} = \frac{30}{12}$$

$$\frac{30}{12} \div 2 = \frac{15}{6}$$