

Use Variables and Algebraic Symbols *notes*

Vocabulary		
Term	Definition	Example
Variables	A letter that represents a number whose value may change	x, y, z The "x" in $2x + 3 = 5$
Constants	A number whose value always stays the same	0, 1, 2, 3, 4, 5...

Using Variables and Symbols in Algebra

Let's say that Lily is 5 and Joe is 12. You know that Lily is 6 years younger than Joe. No matter what Lily's age, Joe will always be 6 years older, and no matter how old Joe is, Lily will always be 6 years younger.

In the language of algebra, we say that Lily's age and Joe's age are **variables**, and the 6 is a **constant**. The ages change ("vary") but the 6 years between them always stay the same ("constant").

In algebra, we use letters of the alphabet to represent **variables**. We could call Lily's age L and Joe's age J , then we could use $J - 6$ to represent Lily's age.

The letters we used to represent the changing ages are called **variables**, and the most commonly used letters to represent variables are $x, y, a, b,$ and c .

Writing Algebraically

To write algebraically, we need operation symbols as well as numbers and variables. These operations are the ones you have seen all through elementary and middle school! Some of them have new symbols that you may or may not have seen before.

Operation	Notation	Say:	The result is...
Addition	$a + b$	a plus b	The sum of a and b
Subtraction	$a - b$	a minus b	The difference of a and b
Multiplication	$a \cdot b$; ab ; $(a)(b)$; $(a)b$; $a(b)$	a times b	The product of a and b
Division	$a \div b$; a/b ; $b\overline{)a}$	a divided by b	The quotient of a and b, where a is the dividend and b is the divisor.

Writing Algebraically continued...

When translating from symbolic form to English, or from English to symbolic form, pay attention to the words “of” and “and.”

- The *difference of 8 and 3* means to subtract 8 and 3, or in other words, 8 minus 3, which we would write as $8 - 3$.
- The *product of 8 and 2* means to multiply 8 and 2, or in other words 8 times 2, which we would write as $8 \cdot 2$.

One thing to note: in algebra, the cross symbol, \times , is not used to show multiplication because that symbol may cause confusion. To make it clearer, we use \cdot or parentheses to indicate multiplication.

Just a few more things...

- When two quantities have the same value, we say they are equal and connect them with an equal sign.
- $a = b$ is read as “a is equal to b”
- On the number line, the numbers get larger as they go from left to right.
- $a < b$ is read as “a is less than b”; a is to the left of b on the number line
- $a > b$ is read as “a is greater than b”; a is to the right of b on the number line.
- \neq means “not equal to”
- \leq means “less than or equal to”
- \geq means “greater than or equal to.”

Example

Translate from algebra to English:

Ⓐ $17 \leq 26$

Ⓒ $12 > 27 \div 3$

Ⓑ $8 \neq 17 - 3$

Ⓓ $y + 7 < 19$

your turn

Translate from algebra to English:

Ⓐ $14 \leq 27$

Ⓒ $12 > 4 \div 2$

Ⓑ $19 - 2 \neq 8$

Ⓓ $x - 7 < 1$

Vocabulary

Term	Definition	Example
Grouping Symbols	Help make clear which expressions are to be kept together and separate from other expressions.	Parentheses () Brackets [] Braces { }
Expression	A number, a variable, or a combination of numbers and variables using operation symbols. <i>Does NOT have an equal sign</i>	$3 + 5$ $n - 1$ $6 \cdot 7 + 8$
Equation	Two expressions linked with an equal sign.	$3 + 6 = 9$ $X - 8 = 4$ $z - 4 = 3z + 5$

Examples

Determine if each is an expression or an equation:

Ⓐ $2(x + 3) = 10$

Ⓑ $4(y - 1) + 1$

Ⓒ $x \div 25$

Ⓓ $y + 8 = 40$

your turn

Determine if each is an expression or an equation:

$3(x - 7) = 27$

$5(4y - 2) - 7$

$y^2 \div 14$

$4x - 6 = 22$

Name: _____ Date: _____ Period: _____

Exponents *notes*

Exponents

Suppose we need to multiply 2 nine times. We could write this as
 $2 \cdot 2 \cdot 2$.

This is tedious and it can be hard to keep track of all those 2s, so we use exponents.
 We write $2 \cdot 2 \cdot 2$ as 2^3 and $2 \cdot 2 \cdot 2$ as 2^9 .

In expressions such as 2^3 , the 2 is called the *base* and the 3 is called the *exponent*. The exponent tells us how many times we need to multiply the base.

We read 2^3 as "two to the third power" or "two cubed." base \rightarrow 2^3 \leftarrow exponent

We say 2^3 is in *exponential notation* and $2 \cdot 2 \cdot 2$ is in *expanded notation*.

Anything to the zero power is equal to 1.

Example

Simplify:

3^4

5^6

x^2

your turn

Simplify:

5^3

1^7

7^2

0^5

Simplifying Expressions Using Order of Operations *notes*

Simplifying Expressions

To **simplify an expression** means to do all the math possible.

For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. I like to work down the page, writing each step of the process below the previous step to keep things organized. The example just described would look like this:

$$\begin{array}{r} 4 \cdot 2 + 1 \\ 8 + 1 \\ 9 \end{array}$$

By not using an equal sign when you simplify an expression, you may avoid confusing expressions with equations.

Order of Operations

Let's take a moment and review the Order of Operations.

HOW TO SIMPLIFY WITH THE ORDER OF OPERATIONS

Parentheses and Other Grouping Symbols	Simplify all expressions inside the parentheses or other grouping symbols, working from the inside out
Exponents	Simplify all expressions with exponents
Multiplication & Division	Perform all multiplication and division in order from left to right. They have equal priority.
Addition and Subtraction	Perform all addition and subtraction in order from left to right. They have equal priority.

Example

$$70 \div 10 + 4(6 - 2)$$

$$5 + 23 + 3[6 - 3(4 - 2)]$$

Name: _____ Date: _____ Period: _____

Order of Operations *Practice*

ORDER OF OPERATIONS

Parentheses**P**lease**E**xponents**E**xcuse**M**ultiplication & **D**ivision**M**y **D**ear**A**ddition & **S**ubtraction**A**unt **S**ally

Simplify.

1) $4 + 3 \cdot 7$

2) $(12 - 5) \cdot 7$

3) $8 + 3 \cdot 9$

4) $18 \div 6 + 4(5 - 2)$

5) $30 \div 5 + 10(3 - 2)$

6) $80 \div 10 + 5(9 - 2)$

7) $9 + 53 - [4(9 + 3)]$

8) $72 - 2[4(5 + 1)]$

Evaluating Expressions *notes*

Evaluating Expressions

In the last few examples, we simplified expressions using the order of operations. Now we'll evaluate some expressions—again following the order of operations. To **evaluate an expression** means to find the value of the expression when the variable is replaced by a given number.

To evaluate an expression, substitute that number for the variable in the expression and then simplify the expression.

Example

Evaluate $7x - 4$, when

Ⓐ $x = 5$

Ⓑ $x = 1$

Evaluate the following for $x = 4$, when

Ⓐ x^2

Ⓑ 3^x

Evaluate $2x^2 + 3x + 8$ when $x = 4$

your turn

Evaluate the given function.

$8x - 3$, when Ⓐ $x = 2$ and Ⓑ $x = 1$

$4y - 4$, when Ⓐ $y = 3$ and Ⓑ $y = 5$

$x = 6$, when Ⓐ x^3 Ⓑ 2^x

$2x^2 + 3x + 8$ when $x = 5$

$3x^2 + 4x + 1$ when $x = 3$

$6x^2 - 4x - 7$ when $x = 2$

Identify and Combine Like Terms *notes*

Identify Coefficients

Algebraic expressions are made up of terms. A **term** is a constant, or the product of a constant and one or more variables.

Examples of terms are 7, y , $5x^2$, $9a$, and b^5 .

The constant that multiplies the variable is called the **coefficient**.

Think of the coefficient as the number in front of the variable. The coefficient of the term $3x$ is 3. When we write x , the coefficient is 1, since $x = 1 \cdot x$.

Example

Identify the coefficient of each term:

Ⓐ $14y$

Ⓑ $15x^2$

Ⓒ a

Identify Like Terms

Some terms share common traits. Look at the following 6 terms. Which ones seem to have traits in common?

$5x$ 7 n^2 4 $3x$ $9n^2$

The 7 and the 4 are both constant terms.

The $5x$ and the $3x$ are both terms with x .

The n^2 and the $9n^2$ are both terms with n^2 .

When two terms are constants or have the same variable and exponent, we say they are **like terms**.

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- n^2 and $9n^2$ are like terms.

Example

Identify the like terms:

y^3 , $7x^2$, 14 , 23 , $4y^3$, $9x$, $5x^2$

Identify and Combine Like Terms *Practice*

your turn

Identify the coefficient of each term:

Ⓐ $17x$

Ⓑ $41b^2$

Ⓒ z

Ⓐ $9p$

Ⓑ $13a^3$

Ⓒ y^3

Identify the like terms:

$9, 2x^3, y^2, 8x^3, 15, 9y, 11y^2.$

$4x^3, 8x^2, 19, 3x^2, 24, 6x^3$

Identify the terms in the expression:

$4x^2 + 5x + 17$

$5x + 2y$

Simplify the expression:

$3x^2 + 7x + 9 + 7x^2 + 9x + 8$

$4y^2 + 5y + 2 + 8y^2 + 4y + 5$

Translate to an Algebraic Expression *notes*

Translate an English Phrase to an Algebraic Expression

In the last section, we listed many operation symbols that are used in algebra and then we translated expressions and equations into English phrases and sentences. Now we'll reverse the process. We'll translate English phrases into algebraic expressions. The symbols and variables we've talked about will help us do that.

Operation	Say:	Expression
Addition	a plus b The sum of a and b a increased by b b more than a The total of a and b b added to a	$a + b$
Subtraction	a minus b the difference of a and b a decreased by b b less than a b subtracted from a	$a - b$
Multiplication	a times b the product of a and b twice a	$a \cdot b$; ab ; $(a)(b)$; $(a)b$; $a(b)$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a \div b$; a/b ; $b\overline{)a}$

Example

Translate each English phrase into an algebraic expression:

Ⓐ the difference of $17x$ and 5

Ⓑ the quotient of $10x^2$ and 7

Translate an English Phrase to an Algebraic Expression

How old will you be in eight years? What age is eight more years than your age now?

Did you add 8 to your present age?

Eight "more than" means 8 added to your present age.

How old were you seven years ago?

This is 7 years less than your age now.

You subtract 7 from your present age.

Seven "less than" means 7 subtracted from your present age.

Example

Translate each English phrase into an algebraic expression:

a) Seventeen more than y

b) Nine less than $9x^2$

c) five times the sum of m and n

d) the sum of five times m and n .

e) The length of a rectangle is 6 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

f) June has dimes and quarters in her purse. The number of dimes is three less than four times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Translate to an Algebraic Expression Practice

your turn

Translate each English phrase into an algebraic expression.

The difference of $24x^2$ and 15

The sum of $17y^2$ and 20

The quotient of $14x$ and 9

The product of 9 and b

Eleven more than x

Sixteen less than $14y$

Four times the sum of a and b

The sum of four times a and b

The length of a rectangle is 9 less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Laura has dimes and quarters in her purse. The number of dimes is two more than twelve times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Unit 1.2: **USE THE LANGUAGE OF ALGEBRA** Practice

Translate from algebra to English

1. $16 - 9$

2. $y - 1 > 6$

Determine if each is an expression or an equation:

3. $9 \cdot 6 = 54$

4. $5 \cdot 6 + 3$

5. $x + 10$

6. $z + 7 = 29$

Simplify each expression:

7. 5^3

8. 2^7

9. $(2 + 5) \cdot 6$

10. $2^3 - 12 \div (9 - 6)$

11. $20 \div 5 + 6 \cdot 9$

12. $4^2 + 8^2$

13. $3(2 + 9 \cdot 7) - 4^2$

14. $2[1 + 3(10 - 2)]$

Unit 1.2 : **USE THE LANGUAGE OF ALGEBRA** Practice

Evaluate the following expressions.

15. $7x + 8$
when $x = 2$

16. x^5 when $x = 2$

17. $(x - y)^2$
when $x = 9$, $y = 6$

18. $2x + 2y$
when $x = 18$ and $y = 14$

Identify the coefficient of each term.

19. $8a$

20. $5r^5$

21. xy

Identify the like terms.

22. x^3 , $8x$, 4 , $8y$, 5 , $8x^3$

23. $9a$, a^2 , 16 , $16b^2$, 4 , $9b^2$

Identify the terms in each expression.

24. $14x^2 + 7x + 3$

25. $9y^3 + 7y + 5$

Simplify the following expressions by combining like terms.

26. $10x + 4x$

27. $8d + 7 + 2d + 8$

28. $10a + 9 + 5a - 3 + 7a - 5$

29. $3x^2 + 12x + 11 + 14x^2 + 7x + 6$

Unit 1.2 : USE THE LANGUAGE OF ALGEBRA Practice

Translate the phrases in algebraic expressions.

30. The difference of 15 and 8

31. The product of 6 and 9

32. The quotient of y and 3

33. The sum of $13x$ and $4x$

34. Sam has jazz and classical CDs in his car. The number of jazz CDs is 4 more than the number of classical CDs. Let c represent the number of classical CDs. Write an expression for the number of jazz CDs.

35. Joe has \$5 and \$10 bills in his wallet. The number of fives is four more than six times the number of tens. Let t represent the number of tens. Write an expression for the number of fives.

Answer each question.

36. Explain the difference between an expression and an equation.

37. Explain how you identify the like terms in the expression $9a^2 + 5a + 8 - a^2 - 2$.